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A Comparative Study of Controllers for a Two Wheeled Self-balancing Robot

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ABSTRACT: Two wheeled self –balancing robot is an inverted pendulum concept. It is an unstable, nonlinear and strong coupling system.This work mainly deals with the study of various controllers on a two wheeled self-balancing robot. The model is builded on the basis of state space design model of cart and pendulum system. The basic aim of the work is to originate an efficient controller to act robot in a real time environment. In this paper position control and angle control of the system are evaluated and compared by using LQG controller and an H-infinity controller.Simulation results shows that H infinity controller has superior performance to output feedback and also shows better system’s robustness significantly.

KEYWORDS: Linear Quadratic Gaussian (LQG) controller,H-infinity ($\boldsymbol{H}_{\infty}$) mixed sensitivity controller.

I.INTRODUCTION

Two wheeled self-balancing robot is a complicated non- linear system. It has also become great consideration as a research entity because of the unstable character of the system. The two wheeled self-balancing robot is based on the fundamental principle of Inverted pendulum. Inverted pendulum has many practical applications such as human walking robots, missile launchers, earthquake resistant building design etc. Development of control system for a two wheeled self-balancing robot has been a huge area of research for the past few years. This is mainly due to its nonlinear dynamics [1]. It became an important test platform for the design and development of missiles, automobiles, space crafts, robots. The simplest method of controlling a system is by using a PID controller. In [1], Osama Jamil proposed a Dual-PID and LQR method. The method gives a better response as compared with PI control.

Two wheeled self-balancing vehicle based on the concept of an inverted pendulum is built by researchers at the industrial electronics laboratory. SEGWAY PT is such a one machine developed by Dean Kamen, now commercially obtainable as a battery-powered electric vehicle in the market. Researchers and engineers are working to develop techniques to make a dynamically stable system and to guarantee desired performance and robust solution. Many methods are applied and tested on this system platform. Dual-PID and LQR control techniques are designed and tested in Simulink and analysed for vertical balance and position control [1]. There are many past studies about the stabilization and optimization of two-wheeled inverted pendulum robots. They are state feedback control with pole placement method [2], Proportional-Integral- Derivative (PID) and Proportional-Derivative(PD) controllers, LQR [1], [3] , Model Predictive Control (MPC) . Kalman filtering and PID algorithm is used for a two wheeled car [4]. PI control is not satisfactory for a two wheeled self-balancing robot to act in a real time application. Different new research works has found on inverted pendulum techniques in the implementation of bipedal locomotion [6], [7].

This paper presents LQGand H-infinity mixed sensitivity design for a two wheeled self-balancing robot. Section two presents system modeling. Section three presents the control techniques. The simulation results are discussed in section four. Conclusions of the work are drawn in section five.

II.SYSTEM MODEL AND ASSUMPTIONS

A two-wheeled self-balancing robot mainly composed of two wheels connected to a body frame holding the motor drive, the power and control electronics parts and a battery part. Fig.1 shows LEGO Mindstorm based on an inverted pendulum robot.

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Fig.1 Inverted Pendulum based two-wheeled self -balancing Robot

System modeling is divided mainly into three parts as follows:

- Model of a DC motor.
- Wheels model
- Chassis model

A. Model of a DC motor:

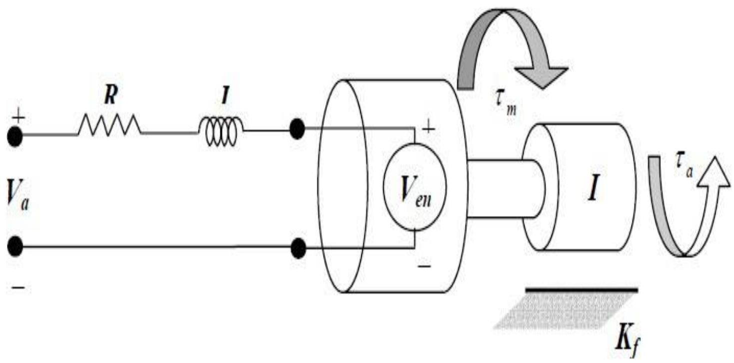

Fig.2A DC motor circuit.

Torque τ_m produced by motor is proportional to current:

$$\tau_m = iK_m \tag{1}$$

Voltage can be expressed as a linear function of velocity of shaft:

$$V_e = K_e\omega \tag{2}$$

According to Kirchoff's law of voltage:

$$L\frac{di}{dt} + Ri = V_a - V_e \tag{3}$$

The equation is obtained as follows:

$$\dot{\omega} = -\frac{K_m K_e \omega}{I_R R} + \frac{K_m V_a}{I_R R} - \frac{\tau_a}{I_R} \tag{4}$$

Matrix form of the equation is as follows:

$$\begin{bmatrix}\dot{\theta}\\ \dot{\omega}\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & \frac{-K_{m}K_{e}}{I_R R}\end{bmatrix}\begin{bmatrix}\theta\\ \omega\end{bmatrix} + \begin{bmatrix}0 & 0\\ \frac{K_m}{I_R R} & \frac{-1}{I_R}\end{bmatrix}\begin{bmatrix}V_a\\ \tau_a\end{bmatrix} \tag{5}$$

$$y = [1 \quad 0]\begin{bmatrix}\theta\\ \omega\end{bmatrix} + [0 \quad 0]\begin{bmatrix}V_a\\ \tau_a\end{bmatrix} \tag{6}$$

B. Model for the wheels of the robot:

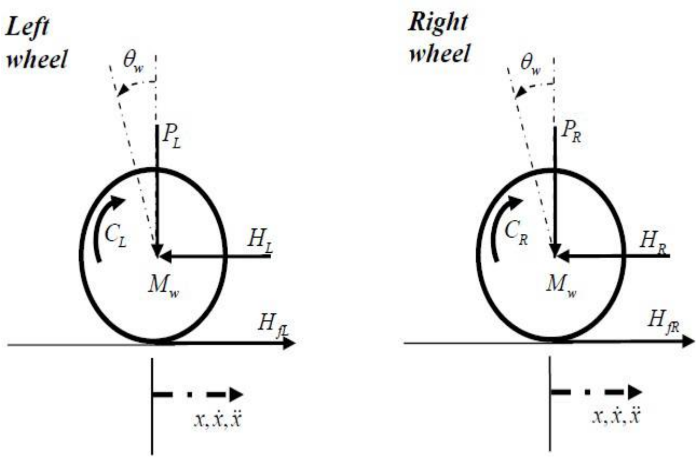

Fig. 3 Free body representation of wheels

Figure illustrates the free body representation of wheels. H_L, H_R, P_L, P_R are the reaction forces acting on the various regions of the body. C_L and C_R are the applying torque to the corresponding wheel.
Left wheel and right wheel equations are obtained as:

For left wheel:

$$M_w\ddot{x} = -\frac{K_{m}K_{e}}{Rr}\dot{\theta}_w + \frac{K_m}{Rr}V_a - \frac{I_w\ddot{\theta}_w}{r} - H_L \tag{7}$$

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For right wheel:

$$M_w\ddot{x} = -\frac{K_{mK_e}}{Rr}\dot{\theta}_w + \frac{K_m}{Rr}V_a - \frac{I_w\ \ddot{\theta}_w}{r} - H_R \qquad (8)$$

Angular rotation can be transformed into linear motion and equation is obtained as follows:

For left wheel:

$$M_w\ddot{x} = -\frac{K_{mK_e}}{Rr^2}\dot{x} + \frac{K_m}{Rr}V_a - \frac{I_{w\ddot{x}}}{r^2} - H_L \qquad (9)$$

For right wheel:

$$M_w\ddot{x} = -\frac{K_{mK_e}}{Rr^2}\dot{x} + \frac{K_m}{Rr}V_a - \frac{I_{w\ddot{x}}}{r^2} - H_R \qquad (10)$$

Summing last two equations of the wheels:

$$2(M_w + \frac{I_w}{r^2})\ddot{x} = -2\frac{K_{mK_e}}{Rr^2}\dot{x} + 2\frac{K_m}{Rr}V_a - (H_L + H_R) \qquad (11)$$

C. Chassis model

Chassis model of the robot is similar to the model of inverted pendulum. After calculating all the equations of the chassis, we will get the following equations:

$$(I_P + l^2M_P)\ddot{\theta}_P - 2\frac{K_{mK_e}}{Rr}\dot{x} + 2\frac{K_m}{R}V_a + M_Pgl\sin\theta_P = -M_P\ddot{x}l\cos\theta_P \qquad (12)$$

$$2\frac{K_m}{Rr}V_a = \left(2M_w + 2\frac{I_w}{r^2} + M_P\right)\ddot{x} + 2\frac{K_{mK_e}}{Rr^2}\dot{x} + M_Pl\ddot{\theta}_P\cos\theta_P - M_Pl\dot{\theta}^2{}_P\sin\theta_P \qquad (13)$$

Equations can be linearized by supposing $\theta_P = \pi + \varphi$, where φ is a small angle with respect to upright direction.
$\cos\theta_P$ = -1, $\sin\theta_P$ = - φ, $\frac{d\theta_P{}^2}{dt}$ = 0

$$\ddot{x} = 2\frac{K_mK_e(M_Plr - I_P - M_Pl^2)}{Rr^2\alpha}\dot{x} + \frac{M_P{}^2gl^2\varphi}{\alpha} + 2\frac{K_m(I_P + M_Pl^2 - M_Plr)}{Rr\alpha}V_a \qquad (14)$$

$$\ddot{\varphi} = 2\frac{K_mK_e(r\beta - M_Pl)}{Rr^2\alpha}\dot{x} + \frac{M_P gl\beta\varphi}{\alpha} + 2\frac{K_m(M_Pl - r\beta)}{Rr\alpha}V_a \qquad (15)$$

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III.CONTROLLER DESIGN

When the system is simulated to an impulse, the controller will try to keep the two wheeled self-balancing robot upright. The design specifications are as follows:

- Settling time for position and angle should be below or equal to 5 seconds.
- With respect to vertical position the angle must be within the limit of ±0.7 radians.

A. Linear Quadratic Gaussian (LQG) Controller

The LQG controller is simply the combination of Kalman filter with anLQR controller. LQG controllers can be used both in linear time-invariant systems as well as in linear time-variant systems.LQG works on separation principle. Here we can design Kalman filter and LQR controller separately. To obtain optimal control, the plant is linearized about the origin to form a continuous time linear system is given by

$$\dot{x}=Ax + Bu \quad (16)$$
$$y = Cx \quad (17)$$

For the optimal control approach is to find a state feedback law that minimize the cost functional

$$J = \frac{1}{2}\int_0^\infty (x^T Qx + U^T RU)dt \quad (18)$$

Where Q is a symmetric positive semi-definite matrix and R is a symmetric positive definite matrix. Q matrix includes the states and their weights whereas R is a scalar matrix.
The optimal control input can be given as:

$$U = -Kx \quad (19)$$

Where, K is the state feedback gain

$$K = R^{-1}B^T P \quad (20)$$

Where P can be evaluated by using the Algebraic Riccati Equation (ARE):

$$A^T P - PA + Q - PBR^{-1}B^T P = 0 \quad (21)$$

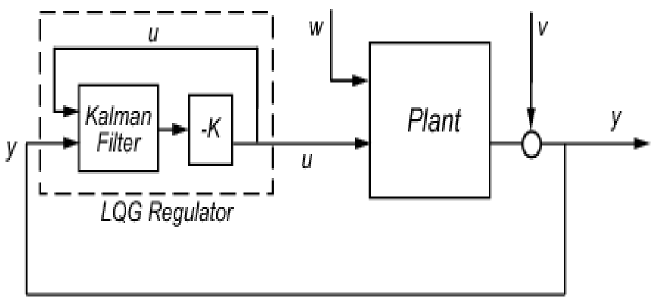

Fig.4Block diagram of LQG controller along with plant.

The state-space representation of optimal compensator (LQG), for regulating the plant with state-space model is given by following state equation:

$$\dot{x}(t)=Ax(t) + BU + L(y(t) - Cx(t) - DU) \quad (22)$$

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$$U = -Kx(t) \quad (23)$$

$$\dot{x}(t)=(A - BK - LC + LDK)x(t) + Ly(t) \quad (24)$$

Where, L is the Kalman Filter gain parameter.

B. H-infinity (H_∞) Controller

H-infinity control is found to guarantee better performance and robustness. H-infinity mixed sensitivity is an important loop shaping method.

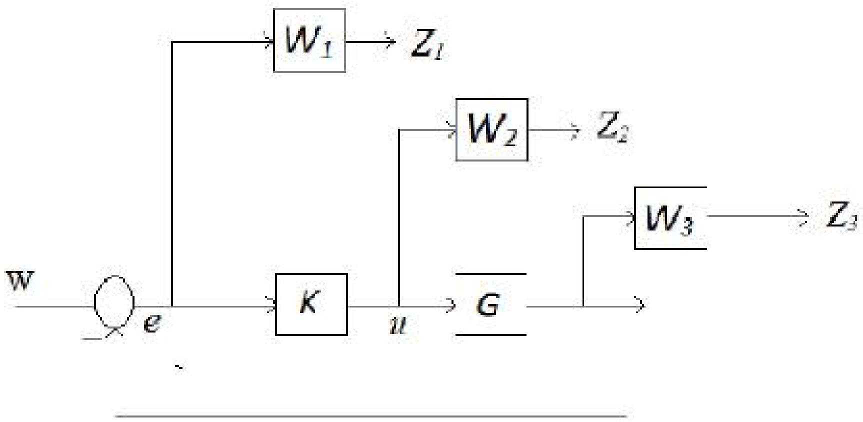

Fig.5 Feedback system with single weighting

With consider to the H-infinity closed norm,the basic requirement for robust stability and performance are depends upon an ultimate one requirement.

$$\|T_{ZR}\| < 1 \quad (25)$$

$$min\|P\|=\min\begin{bmatrix} W_1S \\ W_2KS \\ W_3T \end{bmatrix}=\gamma \quad (26)$$

$$\|T_{ZR}\| = \gamma \quad (27)$$

Where W_1 and W_3 are the weights to be designed by the designer. Weights are mainly used to shape S, R and T.

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IV. RESULT AND DISCUSSION

In the fig 6, it shows the graph of position and angle Vs time. Settling time for position is about 3.25s and settling time for angle is about 3.06s.Angle is about 0.0017 rad from the vertical.

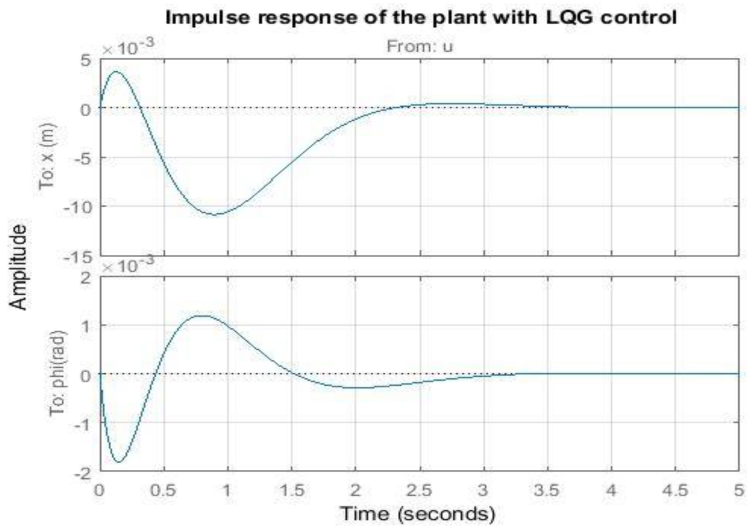

Fig.6 Impulse response of the robot system with LQG control

In fig 7, shows position Vs time response with H-infinity controller.Settling time for position is about 2.91s.

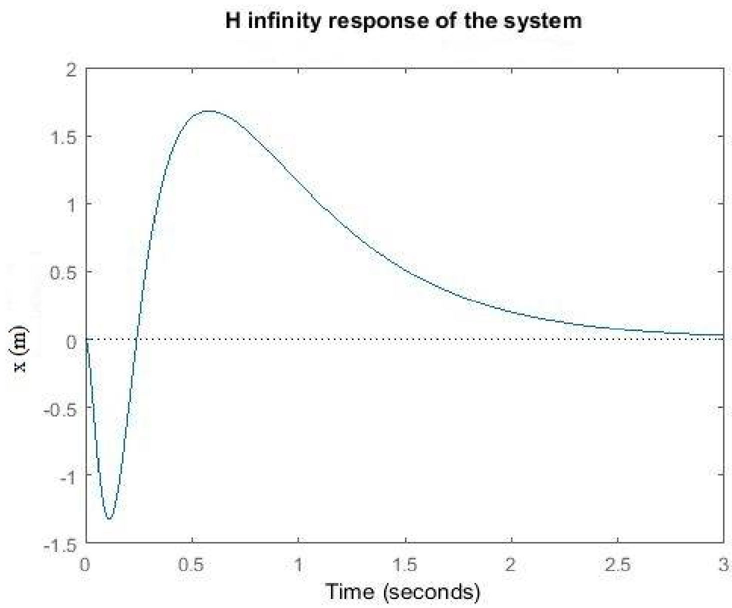

Fig.7 Impulse response of position of the robot system with H-infinity controller

In fig 8, shows angle Vs time responses H-infinity controller .Settling time for angle is about 2.41s.Angle is about 0.632 rad from the vertical.

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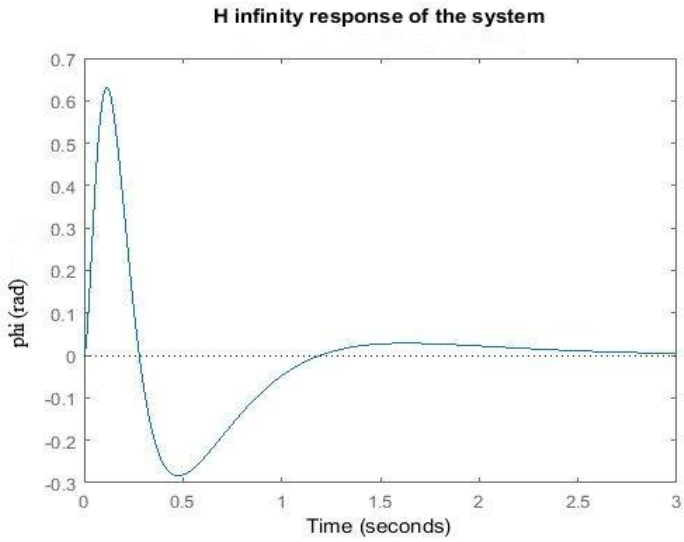

Fig.8Impulse response of angle of the robot system with H-infinity controller

In fig 9, shows singular values Vs frequency response. For better performance $\frac{1}{\sigma(S)}$must lie above the performance bound. For better robust stability $\sigma(T)$ must lie below the bound of robustness. γ Obtained is less than 1 that is 0.062.

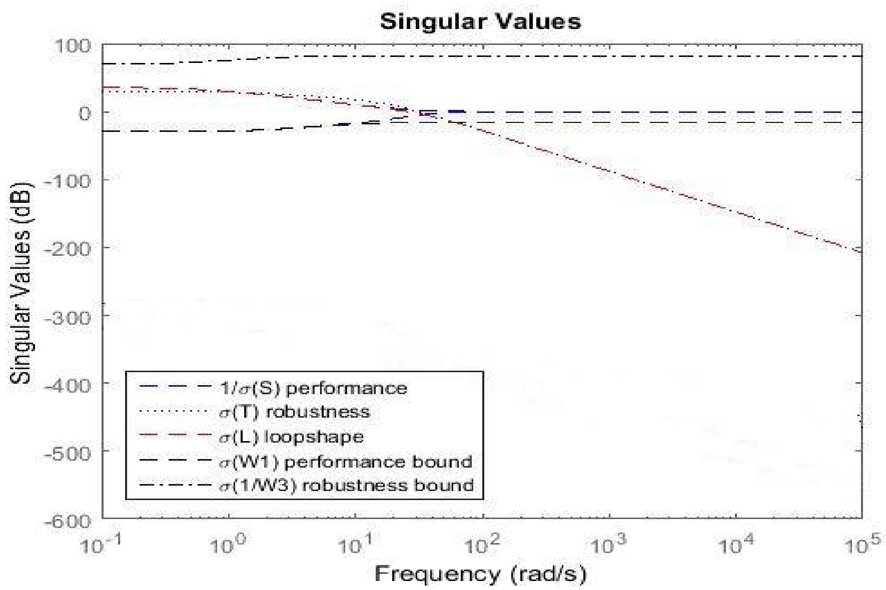

Fig.9 singular values Vs frequency

V.CONCLUSION

Simulations were carried out in MATLAB. The results of impulse response of the system with the LQG controller and H-infinity mixed sensitivity controller are evaluated and compared. Both controller results were satisfactory with the design specifications.But it was found that the designed H-infinity mixed sensitivity controller exhibits a much better performance and robustness as compared to LQG controller.

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REFERENCES

[1] Osama Jamil, Mohsin Jamil, Yasar Ayaz, and Khubab Ahmad, "Modeling, Control of a Two-Wheeled Self-Balancing Robot", International Conference on Robotics and Emerging Allied Technologies in Engineering (iCREATE), Vol.10, pp. 191 - 199, 2014.

[2] S. W. Nawawi, M. N. Ahmad, and J. H. S. Osman, "Real-Time Control of a Two-Wheeled Inverted Pendulum Mobile Robot", World Academyof Science, Engineering and Technology, Vol.39, pp. 214-220, 2008.

[3] A.N.K. Nasir, M.A. Ahmad, and R.M.T. Raja Ismail, "The control of a highly non-linear two-wheels balancing robot: A comparative assessment between LQR and PID-PID control schemes", Optimization, Vol.506, pp.12720, 2010.

[4] Liu Kun, Bai Ming, and Ni Yuhua, "Two-wheel self-balanced car based on Kalman filtering and PID algorithm", 2011 IEEE 18Th International Conference on Industrial Engineering and Engineering Management (IE&EM), Vol.Part 1, pp.281 – 285, 2011.

[5] K.Pathak,J.Franch, and S.K.Agrawal,"Velocity and position control of a wheeled inverted pendulum by partial feedback linearization", IEEE Transactions on Robotics, Vol.21 No:3,pp.505-513,2005.

[6] Hooshang Hemami, Frank C. Weimer, and Said H. Koozekanami,"Some Aspects of the Inverted Pendulum Problem for Modeling of Locomotion Systems, IEEE Transaction on Automatic Control", Vol. 18 No: 6, pp. 658-661, 1973.

[7] Yao Li, and William S. Levine, "An Optimal Control Model for Human Postural Regulation", 2009 American Control Conference, pp. 4705 4710, 2009.

[8] Amir A. Bature, salinda Buyamin, Mohamed .N.Ahamad , and Mutapha muhammad, " A Comparison of Controllers for Balancing Two Wheeled Inverted Pendulum Robot", International Journal of Mechanical&Mechatronics Engineering IJMME-IJENS, Vol.14 No:03, pp. 62-68, 2014.